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OPTIMAL PLASTIC DESIGN OF STIFFENED SHELLS

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Abstract—This paper is concerned with the variational formulation of the optimum design of plastic cylindrical shells with stepwise varying thickness and stiffeners at joints (sections between two contiguous elements). The optimality criterion is attained by using a variational formulation. The saving achieved by using stiffeners is shown by means of an example.

1. INTRODUCTION

During recent years, optimal plastic design of cylindrical shells has been studied by several authors either using variational formulations or on the basis of the static and kinematic theorems of limit analysis [1–5].

In the present paper, optimality conditions of cylindrical shells with stepwise varying thickness and stiffeners at joints are obtained by a variational formulation and an application of these optimality conditions is presented. For the derivation of optimality conditions a similar approach has been used for plates and beams by one of the authors and his co-workers[6–10]. The static and kinematic theorems have been used by several authors to the same purpose[11–16].

It can be outlined from the present study that up to 23% of material can be saved by the utilization of stiffeners. Other applications are being studied by one of the authors[17].

2. FORMULATION OF THE PROBLEM

The usual Kirchhoff–Love assumptions of shell theory with small deflections are adopted. Shell material is assumed to be rigid-perfectly plastic with tensile and compressive yield limits of the same magnitude.

Let L, R, T denote total length, mid-surface radius and thickness respectively (Fig. 1). Loads are positive in the outward radial direction, no axial load is considered (Fig. 2). The shell, the loading, as well as the support conditions are axi-symmetric.

A cylindrical coordinate system (r, θ, X) is chosen (Fig. 1). All functions will depend on X only.

Let the shell be made of n cylindrical elements, each of which has a constant thickness $T_i (i = 1, 2, \ldots, n)$ and length $l_i = X_{i+1} - X_i$. Let a stiffener be at each joint (section separating two contiguous elements); thus $(n-1)$ stiffeners are considered (Fig. 3).

The positive directions of axial bending moment M, circumferential normal force N, circumferential bending moment M_θ, and shear force S are defined in Fig. 2.

The equilibrium equations are[14]: for the element between X_i and X_{i-1}

$$M_i'' + \frac{N_i}{R} - P = 0 \quad i = 1, \ldots, n \tag{2.1}$$

for the stiffener at the ith joint (Fig. 3)

$$\Delta S_i + \frac{f_i}{R} = 0 \quad i = 2, \ldots, n \tag{2.2}$$

where f_i is an internal hoop force acting on the stiffener cross-section.

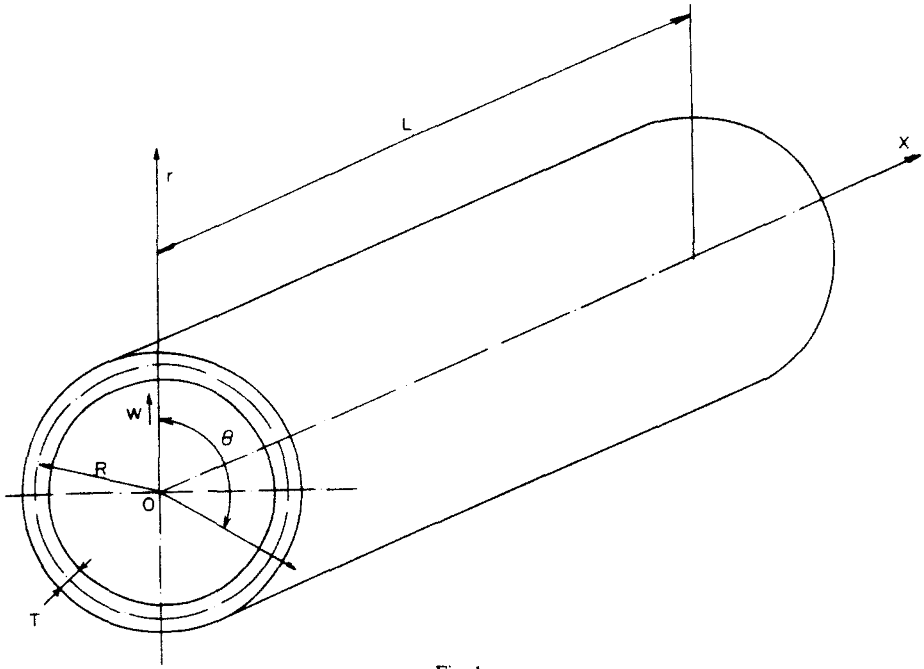

Fig. 1.

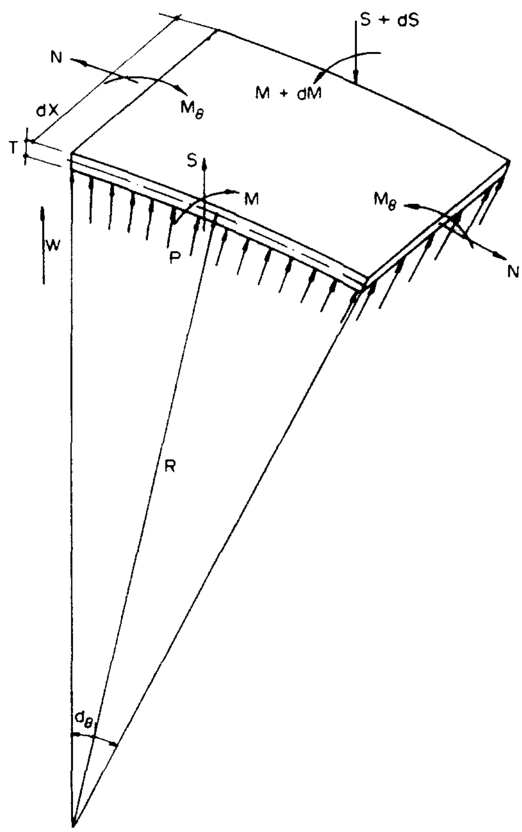

Fig. 2.

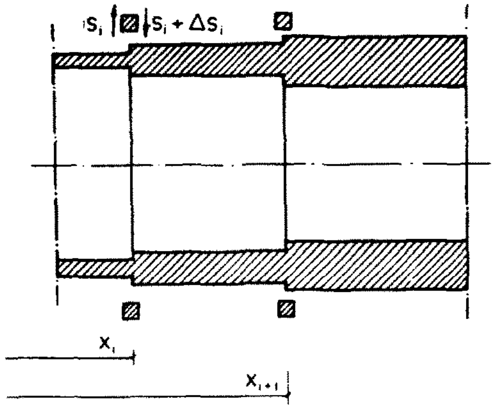

Fig. 3.

The yield conditions are: for the element between X_i and X_{i+1}[14]

$$F(M_i, N_i, T_i) \leq 0 \quad i = 1, \ldots, n \tag{2.3}$$

for the stiffener at the ith joint

$$f_i \leq \sigma_0 A_i \quad i = 2, \ldots, n \tag{2.4}$$

where σ_0 denotes tensile yield limit and A_i is the area of stiffener cross-section.

Any thickness T_i will be subjected to technological constraints:

$$T_i - T_{max} \leq 0 \quad T_{min} - T_i \leq 0 \quad i = 1, \ldots, n. \tag{2.5}$$

The specific cost, that is the cost per unit area of the median surface for the shell and per unit area of cross-section for the stiffeners, will be denoted respectively by $\gamma(T_i)$ and δ. For the sake of simplicity it is assumed that $\gamma(T_i) = kT_i$ and $\delta = k$ (k being a positive constant). The shell's thickness T_i, and the cross-sectional areas A_i for the stiffeners are to be found in such a way that the structure, subjected to the given load P, is at the verge of plastic collapse and has a minimum total cost Z, where

$$Z = 2\pi Rk \left[\sum_{i=1}^{n} (X_{i+1} - X_i)T_i + \sum_{i=2}^{n} A_i\right]. \tag{2.6}$$

In order to complete the problem formulation, static boundary conditions are to be added.

3. OPTIMALITY CRITERION

By applying the Lagrangian multipliers η_i and θ_i, which are not sign-restricted, to the equality constraints (2.1) and (2.2), and the nonnegative multipliers λ_i, μ_i, α_i, β_i to the inequality constraints (2.3)–(2.5) respectively, the functional L is thus constructed:

$$L = k\left[\sum_{i=1}^{n} (X_{i+1} - X_i)T_i + \sum_{i=2}^{n} A_i\right] + \sum_{i=1}^{n}\left[\int_{X_i}^{X_{i+1}} \eta_i\left(-M_i'' - \frac{N_i}{R} + P\right) dX + \int_{X_i}^{X_{i-1}} \lambda_i F(M_i, N_i, T_i)\, dX\right]$$
$$+ \sum_{i=2}^{n}\left[\theta_i\left(\Delta S_i + \frac{f_i}{R}\right) + \mu_i(f_i - \sigma_0 A_i)\right] + \sum_{i=1}^{n} [\alpha_i(T_i - T_{max}) + \beta_i(T_{min} - T_i)]. \tag{3.1}$$

It is assumed that this functional has a saddle point corresponding to

$$\min_{\substack{T_i,\, A_i \geq 0 \\ M_i,\, N_i}} \quad \max_{\substack{\eta_i,\, \theta_i \\ \lambda_i,\, \mu_i,\, \alpha_i,\, \beta_i \geq 0}} \quad L$$

The stationarity conditions for L are necessary for design optimality. Moreover, if the specific cost function is convex, these conditions are also sufficient[11].

The stationarity conditions of L with respect to the Lagrangian multipliers provide not only relations (2.1)–(2.5) but also the equations

$$\lambda_i F(M_i, N_i, T_i) = 0 \quad i = 1, \ldots, n \tag{3.2}$$

$$\mu_i(F_i - \sigma_0 A_i) = 0 \quad i = 2, \ldots, n \tag{3.3}$$

$$\alpha_i(T_i - T_{max}) = 0 \quad \beta_i(T_{min} - T_i) = 0 \quad i = 1, \ldots, n. \tag{3.4}$$

If the differentiation with respect to T_i, M_i, N_i is denoted by the respective subscripts, the stationarity conditions of L with respect to T_i, A_i, N_i, M_i and f_i respectively are as follows

$$k(X_{i+1} - X_i) + \int_{X_i}^{X_{i+1}} \lambda_i F_{,T_i}\, dX + \alpha_i - \beta_i = 0 \quad i = 1, \ldots, n \tag{3.5}$$

$$k - \sigma_0\mu_i \geq 0 \quad A_i(k - \sigma_0\mu_i) = 0 \quad i = 2, \ldots, n \tag{3.6}$$

$$-\frac{\eta_i}{R} + \lambda_i F_{,N_i} = 0 \quad i = 1, \ldots, n \tag{3.7}$$

$$-\eta_i'' + \lambda_i F_{,M_i} = 0 \quad i = 1, \ldots, n \tag{3.8}$$

$$\frac{\theta_i}{R} + \mu_i = 0 \quad i = 2, \ldots, n. \tag{3.9}$$

It must be pointed out that, in the derivation of Relation (3.8), η_i has been assumed to be twice continuously differentiable. The natural boundary conditions at the edges ($X = X_1 = 0$, $X = X_{n+1} = L$) are

$$\eta_i M_i' = \eta_i' M_i = 0. \tag{3.10}$$

At a free edge $M_i = M_i' = 0$, and hence Relation (3.10) does not impose any constraints on η and η'. At a simply supported edge $M_i = 0$ and $\eta = 0$ is required. At a built-in edge there are no restrictions on M and M': Relation (3.10) requires that $\eta = \eta' = 0$. These facts suggest that the Lagrangian multiplier η is proportional to the rate of deflection $\dot{w}$. Let

$$\Delta S_i = M_i' - M_{i-1}' \quad E_i = \eta_i M_i' - \eta_i' M_i - \eta_{i-1} M_{i-1}' + \eta_{i-1}' M_{i-1} + \theta_i(M_i' - M_{i-1}') \quad i = 2, \ldots, n. \tag{3.11}$$

As η_i and M_i are continuous functions, let

$$\bar{M}_i = M_i(X_i) = M_{i-1}(X_i)$$

$$\bar{\eta}_i = \eta_i(X_i) = \eta_{i-1}(X_i).$$

Equation (3.11) can then be written as follows

$$E_i = (\bar{\eta}_i + \theta_i)\Delta S_i + \bar{M}_i(\eta_{i-1}' - \eta_i'). \tag{3.12}$$

In all cases the product $(\bar{\eta}_i + \theta_i)\Delta S_i$ is zero because either $\Delta S_i = 0$ (M' being continuous), or $\bar{\eta}_i + \theta_i = 0$.

If a circle of plastic articulation occurs at $X = X_i$, then the difference $\eta_{i-1}' - \eta_i'$ will not vanish, and E_i will be reduced to the plastic dissipation in that circle. In this case a given value is prescribed for bending moment $\bar{M}_i$, depending on the assumed plastic yield condition.

Relations (3.5) are to be modified: either the term E_{i,T_i} in the ith equation, or the term $E_{i,T_{i-1}}$ in the $(i-1)$th equation is to be added.

With a stiffener at point X_i, ΔS_i does not vanish and so

$$\bar{\eta}_i + \theta_i = 0. \tag{3.13}$$

Substituting Relations (3.9) and (3.13) into (3.6) leads to

$$k - \sigma_0 \frac{\eta_i}{R} = 0 \quad i = 2, \ldots, n. \tag{3.14}$$

Relation (3.14) is the optimality condition for the stiffeners. In a similar way, by substituting Relations (3.7) and (3.8) into (3.5) the optimality condition for the shell elements is obtained. By eliminating k between (3.5) and (3.14) the specific dissipated power in the shell elements is found to be equal to the specific power in the stiffeners.

4. OPTIMALITY CONDITIONS FOR SANDWICH SHELLS

The hexagonal yield condition (Fig. 4), which is the exact yield condition for sandwich shells made of Tresca's material, is used. For a solid shell made of either Tresca's or Mises' material, the hexagonal yield condition represents a safe linear approximation. For a sandwich shell, if H denotes the thickness of the core and T_i (design variable) the thickness of the sheets, then

$$N_{0i} = 2\sigma_0 T_i$$

$$M_{0i} = \sigma_0 H T_i$$

denote full plastic hoop force and full plastic bending moment respectively.

The equations of the sides of the hexagonal yield condition are

$$\frac{M_i}{H} + N_i - 2\sigma_0 T_i \le 0 \tag{a}$$

$$\frac{M_i}{H} - \sigma_0 T_i \le 0 \tag{b}$$

$$\frac{M_i}{H} - N_i - 2\sigma_0 T_i \le 0 \tag{c}$$

$$-\frac{M_i}{H} - N_i - 2\sigma_0 T_i \le 0 \tag{d}$$

$$-\frac{M_i}{H} - \sigma_0 T_i \le 0 \tag{e}$$

$$-\frac{M_i}{H} + N_i - 2\sigma_0 T_i \le 0. \tag{f}$$

(4.1)

Differentiation of Relation (4.1) with respect to T_i, N_i, M_i and substitution into (3.5), (3.7) and (3.8) leads respectively to:

$$k(X_{i+1} - X_i) - \sigma_0 \int_{X_i}^{X_{i+1}} (2\lambda_i^a + \lambda_i^b + 2\lambda_i^c + 2\lambda_i^d + \lambda_i^e$$

$$+ 2\lambda_i^f)\, dX - (\Delta\eta_i' + \Delta\eta_{i+1}')\sigma_0 H + \alpha_i - \beta_i = 0 \quad i = 1, \ldots, n \tag{4.2}$$

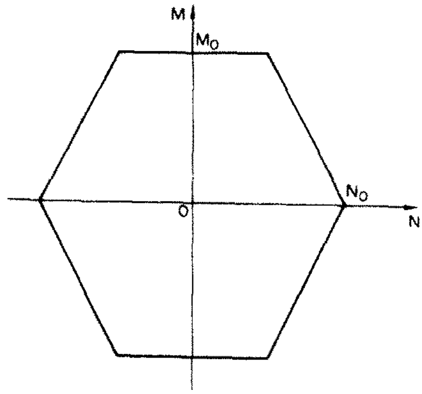

Fig. 4.

$$-\frac{\eta_i}{R}+\lambda_i^a-\lambda_i^c-\lambda_i^d+\lambda_i^f=0$$

$$-\eta''+\frac{1}{H}(\lambda_i^a+\lambda_i^b+\lambda_i^c-\lambda_i^d-\lambda_i^e-\lambda_i^f)=0 \quad i=1,\ldots,n. \tag{4.3}$$

Upper-case indices a to f refer to the corresponding sides of the yield locus. In Relation (4.2) the terms due to boundary dissipations, if any, are introduced by means of the possible discontinuities of η'. If plastic flow occurs on a side, only the corresponding λ is not zero. It can be noticed that Relations (4.3) express the normality rule.

Let D_i denote the specific dissipation in the ith element and D_{si} the specific dissipation in the stiffener at joint i

$$D_i=\frac{\sigma_0}{X_{i+1}-X_i}\int_{X_i}^{X_{i+1}}(2\lambda_i^a+\lambda_i^b+2\lambda_i^c+2\lambda_i^d+\lambda_i^e+2\lambda_i^f)\,dX+(\Delta\eta_i'+\Delta\eta_{i+1}')\frac{\sigma_0 H}{X_{i+1}-X_i} \tag{4.4}$$

$$D_{si}=\sigma_0\frac{\eta}{R}. \tag{4.5}$$

From Relations (3.6), (3.14) and (4.2) if $\alpha_i=\beta_i=0$, that is $T_{min}<T_i<T_{max}$,

(i) with stiffener at the ith joint

$$D_i=D_{si}=k \tag{4.6}$$

(ii) without stiffener

$$D_i=k \quad D_{si}\le k. \tag{4.7}$$

The optimality conditions for the stiffened sandwich shells are clearly pointed out in this way.

5. APPLICATION

The following example aims on one hand at illustrating the application of optimality criterion (4.6) and on the other hand at showing that the use of stiffeners provides a saving of material which may be as much as 23% (Fig. 8).

A simply supported cylindrical sandwich shell is considered with 3 elements, each of which having a constant thickness (Fig. 8). The principle of the method consists in choosing a plastic stress profile on the yield hexagon, so that the equations of Relations (4.3), expressing the normality law, can be integrated. A collapse mechanism is defined in this way, to within some constants which are calculated from both the optimality conditions (4.6) and the kinematic boundary conditions.

With the same stress profile, by substituting N_i from Relation (4.1) into the equilibrium equation (2.1) and subsequently integrating, moment functions M_i are found to within the integration constants and the design variables T_1 and T_2, which are calculated from the static boundary conditions. Let the following dimensionless variables be defined

$$x = \frac{X}{L}, \quad m_i = \frac{M_i}{P_0 RH}, \quad t_i = \frac{2T_i}{P_0 R/\sigma_0}, \quad a_i = \frac{A_i}{LP_0 R/\sigma_0}, \quad \alpha = L\sqrt{\frac{1}{HR}},$$

where P_0 is a uniform outward pressure applied to the shell. In order to find the optimal solution, 3 plastic regimes are used. Owing to the symmetry, only half of the shell is considered (Fig. 8).

(a) In the first solution stiffeners are not considered. A plastic stress profile $A1\ B1\ B2$ (Fig. 5) is assumed. By integrating Relations (4.3) the following expressions are obtained:

$$w_1(x) = \sin \alpha x \quad w_2(x) = a_2 \sin \alpha x + b_2 \cos \alpha x.$$

From the optimality conditions of Relations (4.7) and the following kinematic boundary conditions

$$w_1(0) = 0 \quad w_1(x_1) = w_2(x_1)$$

it follows that

$$x_1 = \frac{1}{3}$$

$$a_2 = \frac{2(1-2x_1)\cos \alpha x_1 + 2x_1(1 - \sin(\alpha/2)\sin \alpha x_1) - \cos 2\alpha x_1}{1 + 2x_1 - 2x_1 \cos \alpha((1/2) - x_1)}$$

$$b_2 = \frac{-2(1-2x_1)\sin \alpha x_1 + 2x_1 \cos(\alpha/2)\sin \alpha x_1 + \sin 2\alpha x_1}{1 + 2x_1 - 2x_1 \cos \alpha((1/2) - x_1)}.$$

By integrating equilibrium equation (2.1), where plasticity conditions (4.1) have been used, the following relation is reached

$$m_i(x) = c_i \sin \alpha x + d_i \cos \alpha x + 1 - t_i \quad i = 1, 2.$$

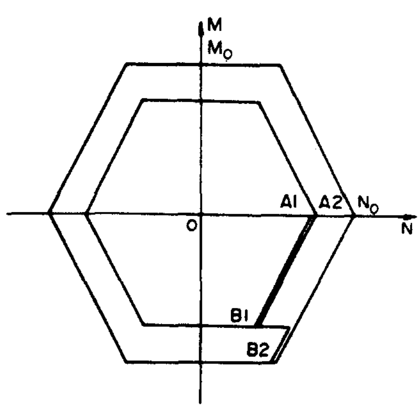

Fig. 5.

From static boundary conditions, the constants c_i, d_i and $t_i (i = 1, 2)$ are calculated

$$t_1 = 2\frac{\cos\alpha((1/2)-2x_1) - 2\cos\alpha x_1 - \cos\alpha((1/2)-x_1) + 2}{\cos\alpha((1/2)-2x_1) - 2\cos\alpha((1/2)-x_1) - 2\cos\alpha x_1 + 4}$$

$$t_2 = 2\frac{\cos\alpha((1/2)-2x_1) - 2\cos\alpha((1/2)-x_1) - 2\cos\alpha x_1 + 3}{\cos\alpha((1/2)-2x_1) - 2\cos\alpha((1/2)-x_1) - 2\cos\alpha x_1 + 4}.$$

The solution here above is both kinematically and statically admissible.

As long as $0 < \alpha < 2.25164$, it can be verified that the dissipation per unit volume of material in a hypothetical stiffener is smaller than the (constant) average dissipation per unit volume in the shell elements. Hence the assumption of vanishing stiffeners is found to be valid, according to Relation (4.7).

At $\alpha = 2.25164$ the dissipation of the stiffener at $x_1 = 1/3$ becomes equal to the dissipation in every shell element.

(b) A plastic stress profile A1B1M1M2B2 (Fig. 6) is chosen: assuming the presence of a stiffener at $x_1 = 1/3$, deflections w_i and moments $m_i (i = 1, 2)$ are obtained

$$w_1(x) = \sin\alpha x \quad 0 \le x \le x^*$$

$$w_2(x) = a_2 \sin\alpha x + b_2 \cos\alpha x \quad x^* \le x \le \frac{1}{2}$$

$$m_1(x) = c_1 \sin\alpha x + d_1 \cos\alpha x + 1 - h_1 \quad 0 \le x \le \frac{1}{3}$$

$$m_2(x) = c_2 \sin\alpha x + d_2 \cos\alpha x + 1 - h_2 \quad \frac{1}{3} \le x \le \frac{1}{2}.$$

Integration constants, design variables and the abscissa x^* of the hinge circle, corresponding to point B2 of the yield hexagon, are calculated from static and kinematic boundary conditions and optimality conditions of Relation (4.6).

Due to transcendental equations, the solution is found by means of numerical computer program. The results are shown in Fig. 8. The considered plastic regime is valid for $2.25164 \le \alpha \le 2.41587$. It is to be emphasized that in this range of α the normality rule is violated at $x = 0.5$. Thus, this solution is only statically admissible: upper bounds of h_1 and h_2 are obtained.

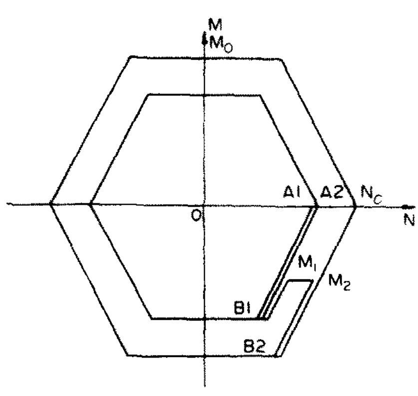

Fig. 6.

(c) Assuming the presence of a stiffener at $x = 1/3$ a plastic stress profile A1B1A1A2 (Fig. 7) is considered. The following optimal deflection and moment fields are obtained

$$w_1(x) = \sin \alpha x \qquad 0 \le x \le \frac{1}{6}$$

$$w_2(x) = a_2 \sin \alpha x + b_2 \cos \alpha x \qquad \frac{1}{6} \le x \le \frac{1}{3}$$

$$w_3(x) = w_0 \qquad \frac{1}{3} \le x \le \frac{1}{2}$$

$$m_1(x) = c_1 \sin \alpha x + d_1 \cos \alpha x + 1 - h_1 \quad 0 \le x \le \frac{1}{3}$$

$$m_2(x) = 0 \qquad \frac{1}{3} \le x \le \frac{1}{2}$$

with

$$w_0 = \frac{-(1/2)\sin(\alpha/3) + 2\sin(\alpha/6)}{(\alpha/3)\sin(\alpha/6) + \cos(\alpha/6) - (1/2)}$$

$$a_2 = \frac{-\sin(\alpha/6)\cos(\alpha/3) + w_0 \cos(\alpha/6)}{\sin(\alpha/6)}$$

$$b_2 = \frac{\sin(\alpha/6)\sin(\alpha/3) - w_0 \sin(\alpha/6)}{\sin(\alpha/6)}$$

$$h_1 = \frac{1 - \cos(\alpha/6)}{1 - (1/2)\cos(\alpha/6)}$$

$$h_2 = 1.$$

The considered plastic regime is valid for $2.41587 < \alpha < 6.99335$ and the solution is kinematically and statically admissible. The second element of the shell is in pure membrane state (Fig. 8).

The solutions are to be completed by finding the volume of the stiffeners which is obtained from the discontinuity of the shear forces

$$a_1 = \frac{1}{\alpha}\left[m_1'\left(\frac{1}{3}\right) - m_2'\left(\frac{1}{3}\right)\right].$$

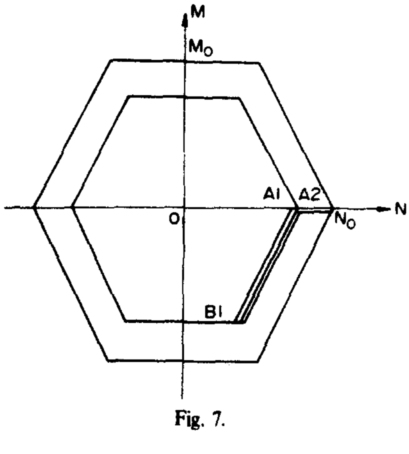

Fig. 7.

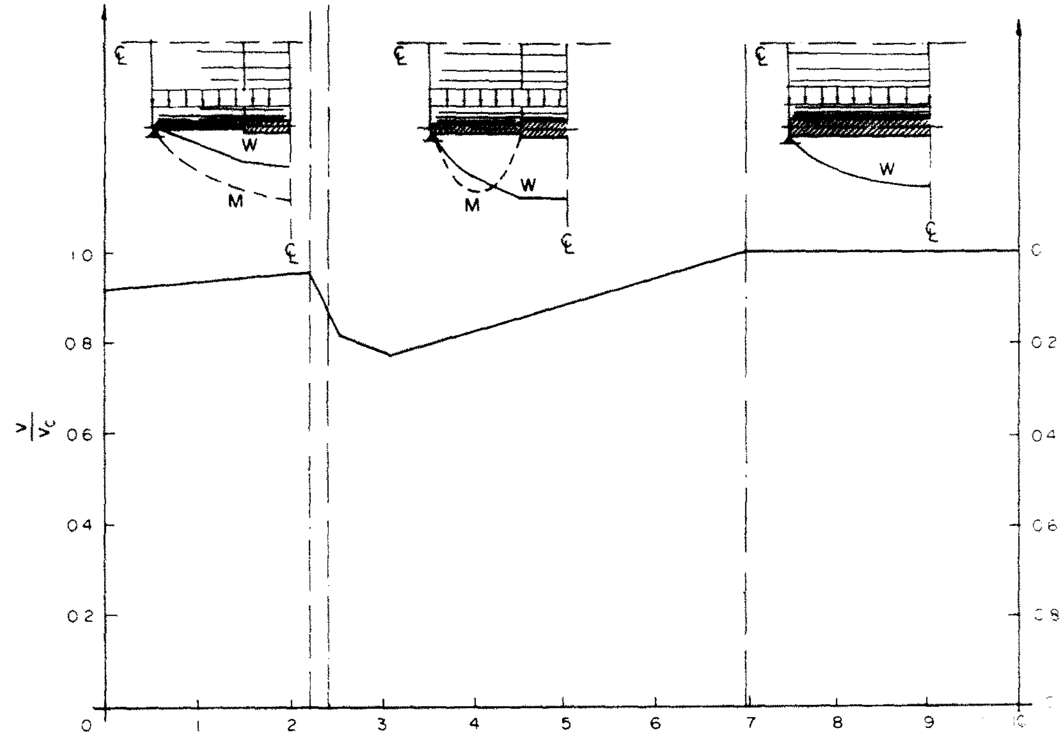

Fig. 8.

In order to evaluate the actual volume saving, the sandwich shell of uniform thickness with the same loading, geometry and supports is now considered. The plastic design is[18]

$$0 \le \alpha \le \pi \ : \ h = \frac{1-\cos(\alpha/2)}{1-(1/2)\cos(\alpha/2)}$$

$$\alpha \ge \pi \ : \ h = 1.$$

The optimal solution saving with respect to the uniform thickness sandwich shell is depicted in Fig. 8. It can be easily verified that the maximum saving of about 22.84% is reached from $\alpha = \pi$. For $\alpha > 6.99335$ no saving can be obtained with respect to a uniform thickness sandwich shell.

6. CONCLUSIONS

Optimality conditions are useful to obtain optimal analytical solutions, the main difficulty being to find plastic regimes that fully solve the given problem. However, numerical solutions can be given by using these optimality conditions[19]; other examples will be presented in Ref.[17]. A saving of about 38% can be reached using optimal sandwich shell which has continuously varying thickness[4]. Therefore, more than 23% of material is expected to be saved by using stiffeners, and by subdividing the shell into more elements.

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